**Phys 410**
**Spring 2013**
**Lecture #16 Summary**
**27 February, 2013**

We finished discussion of the motion of the Foucault pendulum. Defining the constants $\omega_0^2 \equiv g/L$, and $\Omega_z \equiv \Omega\cos\theta$, we get two coupled equations of motion:

$$\ddot{x} - 2\dot{y}\Omega_z + \omega_0^2 x = 0$$

$$\ddot{y} + 2\dot{x}\Omega_z + \omega_0^2 y = 0$$

These can be combined in a manner similar to the equations for motion of a charged particle in a magnetic field. Take the first equation plus "i" times the second equation, and define the new dependent complex variable $\eta(t) \equiv x(t) + iy(t)$ to get a single equation: $\ddot{\eta} + i2\dot{\eta}\Omega_z + \omega_0^2\eta = 0$. Trying a solution of the form $\eta(t) = e^{-i\alpha t}$, we get an auxiliary equation with solutions $\alpha = \Omega_z \pm \sqrt{\omega_0^2 + \omega\Omega_z^2}$. Using the fact that the pendulum oscillates many times compared to the rotation period of the Earth (i.e. $\omega_0 \gg \Omega_z$) we come to the solution $\eta(t) = e^{-i\Omega_z t}\left(C_1 e^{-i\omega_0 t} + C_2 e^{+i\omega_0 t}\right)$. To supply initial conditions, consider pulling the pendulum bob to a displacement $A$ in the east ($x$) direction ($y = 0$) and release it from rest. In this case one finds $C_1 = C_2 = A/2$, and the solution is $\eta(t) = Ae^{-i\Omega_z t}\cos(\omega_0 t)$. Taking the real and imaginary parts to get the actual equations of motion in real space gives $x(t) = A\cos(\Omega_z t)\cos(\omega_0 t)$ and $y(t) = -A\sin(\Omega_z t)\cos(\omega_0 t)$. The pendulum swings back and forth on a short time scale, described by the factor of $\cos(\omega_0 t)$. On longer time scales, the plane of oscillation rotates, as described by the factors of $\cos(\Omega_z t)$ and $-\sin(\Omega_z t)$, with $\omega_0 \gg \Omega_z$. This slow rotation of the plane of oscillation occurs at a frequency that depends on your (co-)latitude on the Earth $\Omega_z \equiv \Omega\cos\theta$, where the rotation frequency of the Earth is $\Omega = 7 \times 10^{-5}$ Rads/s.